Dennis Gaitsgory: The category of singularities as a crystal and global Springer fibers

The series of talks follows the arxiv paper with the same title. The goal is to explain the proof of a certain 'gluing conjecture' on the Galois side of the geometric Langlands correspondence.

In the first talk, I will review the formulation of the geometric Langlands conjecture using ind-coherent sheaves. The current strategy for proving the conjecture involves 'cutting' both sides of the conjecture into more manageable pieces indexed by the conjugacy classes of parabolic subgroups. The strategy relies on two 'gluing statements' on the two sides of the conjecture. I will summarize the strategy and state the gluing conjecture on the Galois side.

In the second talk, we will study the category of singularities on a quasi-smooth scheme (or stack) X. The main idea is that the notion of singular support can be used to equip the category with an additional structure: that of a crystal over a certain projective fibration Y over X (here Y is the projectivization of the shifted cotangent bundle on X).

In the third talk, the crystalline structure will be used to reduce the gluing conjecture to a topological statement. The statement concerns homological contractibility of certain topological spaces obtained by gluing various Springer fibers. I intend to explain the ideas that go into the proof of the statement.