WORKSHOP 'GEOMETRIC LANGLANDS AND DERIVED ALGEBRAIC GEOMETRY'

David Ben-Zvi: Geometric Langlands correspondence and topological field theory

Kapustin and Witten introduced a powerful perspective on the geometric Langlands correspondence as an aspect of electric-magnetic duality in four dimensional gauge theory. While the familiar (de Rham) correspondence is best seen as a statement in conformal field theory, much of the structure can be seen in the simpler (Betti) setting of topological field theory using Lurie's proof of the Cobordism Hypothesis. In these lectures I will explain this perspective and illustrate its applications to representation theory following joint work with Nadler as well as Brochier, Gunningham, Jordan and Preygel.

Dario Beraldo: The extended Whittaker category

In analogy with the classical theory of Whittaker coefficients for automorphic functions, we construct a Fourier transform functor, called coeff_G, from the DG category of D-modules on Bun_G to a certain DG category $\mathrm{Wh}(G, ext)$, called the extended Whittaker category. This construction allows to formulate the compatibility of the Langlands duality functor $\mathbb{L}_G : \mathrm{IndCoh}_N(\mathrm{LocSys}_{\check{G}}) \to D(\mathrm{Bun}_G)$ with the Whittaker model.

For $G = \mathrm{GL}_n$ and $G = \mathrm{PGL}_n$, we prove that coeff_G is fully faithful. This result guarantees that, for those groups, $\mathbb{L}_G$ is unique (if it exists) and necessarily fully-faithful. The proof ultimately relies on the theory of Drinfeld's quasi-maps and on the contractibility of the space of rational maps $X \dashrightarrow \mathbb{P}^n$.

Michael Finkelberg: Towards a cluster structure on trigonometric zastava

We study a moduli problem on a nodal curve of arithmetic genus 1, whose solution is an open subscheme in the zastava space for projective line. This moduli space is equipped with a natural Poisson structure, and we compute it in a natural coordinate system. We compare this Poisson structure with the trigonometric Poisson structure on the transversal slices in an affine flag variety. We conjecture that certain generalized minors give rise to a cluster structure on the trigonometric zastava. This is a joint project with A. Kuznetsov and L. Rybnikov.

Dennis Gaitsgory: The category of singularities as a crystal and global Springer fibers

The series of talks follows the arxiv paper with the same title. The goal is to explain the proof of a certain 'gluing conjecture' on the Galois side of the geometric Langlands correspondence.

In the first talk, I will review the formulation of the geometric Langlands conjecture using ind-coherent sheaves. The current strategy for proving the conjecture involves 'cutting' both sides of the conjecture into more manageable pieces indexed by the conjugacy classes of parabolic subgroups. The strategy relies on two 'gluing statements' on the two sides of the conjecture. I will summarize the strategy and state the gluing conjecture on the Galois side.

In the second talk, we will study the category of singularities on a quasi-smooth scheme (or stack) X. The main idea is that the notion of singular support can be used to equip the category with an additional structure: that of a crystal over a certain projective fibration Y over X (here Y is the projectivization of the shifted cotangent bundle on X).

In the third talk, the crystalline structure will be used to reduce the gluing conjecture to a topological statement. The statement concerns homological contractibility of certain topological spaces obtained by gluing various Springer fibers. I intend to explain the ideas that go into the proof of the statement.

Sam Raskin: QCoh and IndCoh in derived algebraic geometry (preparatory talk for Gaitsgory's mini-course)

Abstract: In the theory of derived categories of coherent sheaves on varieties, one must grapple early on with the difference between perfect complexes and coherent complex (i.e., bounded complexes with coherent cohomologies). If one works instead with categories admitting infinite direct sums, the terms take on new names, and this becomes the distinction between QCoh and IndCoh.

In this talk, we will give an introduction to QCoh and IndCoh. In particular, we will explain how IndCoh is the natural framework for Grothendieck's interpretation of Serre duality, and the role in plays in the theory of D-modules.

Penghui Li: Analytic uniformazation of the semi-stable locus of G-torsors on an elliptic curve.

Let E be an elliptic curve over complex numbers. Motivated by the Betti version of the geometric Langlands for a reductive group G and the curve E, we try to describe the category of analytic sheaves on the semi-stable locus of Bun_G with the nilpotent singular support via the Loojenga uniformazation of this stack.

David Nadler: Betti Langlands in genus one

We will report on an ongoing project to understand geometric Langlands in genus one, in particular a version that depends only on the topology of the curve (as appears in physical descriptions of the subject). The emphasis will be on the realization of the automorphic and spectral categories as the center/cocenter of the affine Hecke category. We will mention work with D. Ben-Zvi and A. Preygel that accomplishes this on the spectral side, then focus on ongoing work with D. Ben-Zvi, building on work with P. Li, that we expect will lead to a parallel automorphic result.

Sam Raskin: Spectral decomposition of the principal series category

We will discuss the problem of Langlands duality for the principal series category (alias: D-modules on the semi-infinite flag variety). In particular, we will explain how to relate Whittaker invariants to local systems for the Langlands dual group. This work can be understood as a chiralization of the Arkhipov–Bezrukavnikov theory.

Nick Rozenblyum: Higher differential operators and applications

One can regard the algebra of differential operators as a 1-dimensional object. I will explain this point of view and describe higher (and lower) dimensional analogues of the algebra of differential operators. For instance, a two dimensional example of this construction produces the vertex algebra of chiral differential operators. I will describe some applications of this construction to geometry and representation theory.

Bertrand Toën: Infinitesimal aspects of derived algebraic geometry

This series of lectures is an introduction to derived algebraic geometry with a particular focus on infinitesimal aspects (differential calculus, etc.). The first lecture will present the general language of derived algebraic geometry: derived schemes and stacks, derived categories, cotangent complexes, derived mapping stacks and representability. It will also include a very short review of basic infinity-category theory (which will be used all along the lectures). The second lecture will be devoted to the de Rham theory of derived algebraic stacks and its relations to formal completions and formal derived stacks: derived de Rham complex, Hodge filtration, relation with Betti cohomology and with derived loop spaces. Finally, the in the last lecture I will introduce shifted symplectic and shifted Poisson structures on general derived algebraic (n-)stacks. I will explain how the materials of the two previous lectures can be used in order to prove the existence of canonical deformation quantizations of shifted Poisson structures. This last lecture includes a glimpse of the theory of infinity-operads and their geometrico-algebraic models, formality theorems as well as open problems.

Zhiwei Yun: A simple case of ramified geometric Langlands

This is joint work with David Nadler. We prove the categorical geometric Langlands correspondence in the following situation: the curve is $\mathbb{P}^1$, the group G is either SL_2 or PGL_2, and the level structure is Iwahori at three points of $\mathbb{P}^1$ and unramified elsewhere.