Spectrum of Random Graphs
January 4 - 8, 2016

Nalini Anantharaman: Quantum ergodicity on large graphs.

With Etienne Le Masson we established quantum ergodicity – a form of delocalization – for eigenfunctions of the laplacian on large regular graphs of fixed degree. The result applies to deterministic sequences of regular graphs, provided they are expanders and converge to a tree in the sense of Benjamini-Schramm (so it applies in particular to random regular graphs). We will sketch several proofs of the result and will discuss the possibility to adapt the methods to other models of interest in mathematical physics, such as the Anderson model on large regular graphs, regular graphs with weighted edges, or possibly certain models of non-regular graphs.

Matthias Keller: Does diffusion determine the geometry of a graph?

Can one hear the shape of a drum?" is a famous question by Mark Kac. This question asks whether the spectrum of the Laplacian determines the underlying domain. However, it is well known that the answer is no in general for domains as well as for graphs. Abstractly the question of Kac is whether a unitary transformation of the Laplacians preserves the geometry. Following ideas of Wolfgang Arendt, we reformulate the question by replacing the unitary transformation by an order isomorphism. This leads to the question whether a transformation of the heat semigroups under order isomorphisms determines the geometry. We address this question for general weighted graphs.
(This is joint work with Daniel Lenz, Marcel Schmidt and Melchior Wirth)

Reimer Kuehn: Spectra of Random Stochastic Matrices and Relaxation in Complex Systems

We compute spectra of large stochastic matrices W, defined on sparse random graphs, where edges (i,j) of the graph are given positive random weights $W_{ij} > 0$ in such a fashion that column sums are normalized to one. We compute spectra of such matrices both in the thermodynami limit, and for single large instances. The structure of the graphs and the distribution of the non-zero edge weights W_{ij} are largely arbitrary, as long as the mean vertex degree remains finite in the thermodynamic limit and the W_{ij} satisfy a detailed balance condition. Wigner semi-circular laws are recovered in the limit of large mean connectivity. Knowing the spectra of stochastic matrices is tantamount to knowing the complete spectrum of relaxation times of stochastic processes described by them, so our results should have many interesting applications for the description of relaxation in complex systems. Our approach allows to disentangle contributions to the spectral density related to extended and localized states, respectively, allowing to differentiate between time-scales associated with transport processes and those associated with the dynamics of local rearrangements.

Mira Shamis: The Wegner orbital model.

TBA

Pierre Youssef: Invertibility of the adjacency matrix of a random digraph

It is conjectured by Vu that the adjacency matrix of a uniform random d-regular graph on n vertices is asymptotically invertible almost surely whenever $d \geq 3$. We consider the directed version of this conjecture and show that there are universal constants c, C such that for any $C \leq d \leq cn/\ln^2 n$, the adjacency matrix of a uniform random directed d-regular graph is invertible with probability at least $1 - C\ln^3 d/\sqrt{d}$. This extends a previous result of Cook who showed a similar statement under the restriction that $d \gg \ln^2 d$.
This is a joint work with A. Litvak, A. Lytova, K. Tikhomirov and N. Tomczak-Jaegermann.