Distances of elements in valued field extensions

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Ordered Algebraic Structures and Related Topics

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notions

(K, v) a valued field, v written in the additive way:

- $v(x) = \infty \Leftrightarrow x = 0$
- $v(xy) = v(x) + v(y)$
- $v(x + y) \geq \min\{v(x), v(y)\}$,

for all $x, y \in K$.

vK the value group,
Kv the residue field,

$\widetilde{vK}$ the divisible hull of vK,
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For every extension $(L|K,v)$ of valued fields and $a \in L$ we define

$$v(a-K) \;:=\; \{v(a-c)\,|\,c \in K\}.$$

- The set $v(a-K) \cap vK$ is an initial segment of vK.

Define the **distance of** a **from** K to be the cut dist $(a,K) := (\Lambda_L, \Lambda_R)$ in $\widetilde{vK}$, where Λ_L is the smallest initial segment of $\widetilde{vK}$ containing

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Proposition 1

Take algebraic extensions $(K(a)|K,v)$, $(K(b)|K,v)$ of prime degree and assume that the valuation v of K extends in a unique way to the fields $K(a)$ and $K(b)$. Assume moreover that $v(b-a) > \operatorname{dist}(a,K)$. Then

$$vK(a) = vK(b) \text{ and } K(a)v = K(b)v.$$

Assume that $(L|K,v)$ is a finite extension, v extends in a unique way from K to L and $p =$charexp Kv. Then by the Lemma of Ostrowski,

$$[L:K] = d(L|K,v)(vL:vK)[Lv:Kv],$$

where $d(L|K,v) = p^n$ for some $n \geq 0$ is the **defect** of $(L|K,v)$.

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Take a defectless extension $(L|K,v)$ of prime degree and assume that the valuation v of K extends in a unique way to L.

Then for every $a \in L$ the set $v(a-K)$ admits a maximal element.

- If $vL = vK$, then the distance of every element $b \in L \setminus K$ from K is of the form

$$\alpha^+ := (\{\beta \in \widetilde{vK} \mid \beta \leq \alpha\}, \{\beta \in \widetilde{vK} \mid \beta > \alpha\})$$

for some $\alpha \in vK$. Conversely, for every $\alpha \in vK$ there is $b \in L \setminus K$ such that $\operatorname{dist}(b,K) = \alpha^+$.

- If the value group extension $vL|vK$ is nontrivial, then the distance of every element $b \in L \setminus K$ from K is of the form α^+ for some $\alpha \in vL \setminus vK$. Furthermore, for every $\alpha \in vL \setminus vK$ there is $b \in L \setminus K$ such that $\operatorname{dist}(b,K) = \alpha^+$.

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An extension $(L|K, v)$ of valued fields is called **immediate** if $[vL : vK] = 1 = [Lv : Kv]$.

Theorem 1

An extension $(L|K, v)$ of valued fields is immediate if and only if for every element $z \in L \setminus K$ the set $v(z - K)$ and has no maximal element.

Take a sequence $(a_\nu)_{\nu<\lambda}$ of elements of K such that $(v(z - a_\nu))_{\nu<\lambda}$ is strictly increasing and cofinal in $v(z - K)$.

- $v(a_\tau - a_\sigma) > v(a_\sigma - a_\rho)$ if $\rho < \sigma < \tau < \lambda$
- $v(z - a_\mu) > v(z - a_\nu)$ if $\nu < \mu < \lambda$.

$(a_\nu)_{\nu<\lambda}$ is called a **pseudo Cauchy sequence in** (K, v), z is called a **pseudo limit** of $(a_\nu)_{\nu<\lambda}$

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- Pseudo Cauchy sequences are a useful tool in particular in a proof of a theorem giving conditions for a valued field to admit immediate extensions of infinite transcendence degree.

A valued field is called **maximal** if it admits no proper immediate extensions.

- Every maximal field (M, v) is henselian and defectless.
- A finite extension of maximal field is again a maximal field.

Theorem 2

Take a maximal field (K, v) of characteristic 0 or of positive characteristic p and finite p-degree. If $(L|K, v)$ is an algebraic extension, then the field (L, v) is maximal if and only if $L|K$ is finite.

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Artin-Schreier defect extensions

Assume that (K, v) is a valued field of characteristic $p > 0$, $K(\vartheta)|K$ an Artin-Schreier defect extension with $\vartheta^p - \vartheta - a = 0$ for some $a \in K$.

- $\operatorname{dist}(\vartheta, K)$ does not depend on the choice of ϑ.

$K(\vartheta)|K$ is called a dependent Artin-Schreier defect extension if there is an immediate purely inseparable extension $K(\eta)|K$ of degree p such that $v(\eta - \vartheta) > \operatorname{dist}(\vartheta, K)$. Otherwise it is called an independent Artin-Schreier defect extension.

Proposition 2

The Artin-Schreier defect extension $K(\vartheta)|K$ is independent if and only if $\operatorname{dist}(\vartheta, K) = H^-$ for some proper convex subgroup H of $\widetilde{vK}$.

$$H^- = (\{\alpha \in \widetilde{vK} \mid \alpha < H\}, \{\alpha \in \widetilde{vK} \mid \exists \beta \in H \ \beta \geq \alpha\})$$

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Applications of the classification of A.-S. defect extensions:

- a characterization of defectless fields of positive char.,
- the structure of maximal immediate extensions of valued fields.

Every valued field admits a maximal immediate extension.

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The structure of maximal immediate extensions

Theorem 3

Take a henselian field (K, v) of positive residue characteristic p with p-divisible value group and perfect residue field. Assume that (K, v) admits a maximal immediate extension of finite transcendence degree. Then

- *(K, v) admits no immediate separable-algebraic extensions,*
- *If $\mathrm{char} K = p$, then $K^{1/p^\infty} \subseteq K^c$,*
- *the maximal immediate extension of (K, v) is unique up to isomorphism.*

Theorem 4

There are valued fields which admit an algebraic maximal immediate extension as well as one of infinite transcendence degree.

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