F an algebraic function field in one variable over $\mathbb{R}$, i.e. $[F:\mathbb{R}(X)]<\infty$

$$\sum_1^k F^n = \{\sum_1^k x_i^n \mid x_i \in F\}, \sum F^n := \bigcup_k \sum_1^k F^n,$$

$p_n(F) = p_n = \min\{k \mid \sum F^n = \sum_1^k F^n\}$, the $n-$th Pythagoras number of F,
$s_n(F) = s_n = \min\{k \mid -1 \in \sum_1^k F^n\}$, the $n-$th stufe (level) of F.

Some results so far:

(1) (Witt, 1934) $p_2 = 2$,
(2) (Richmond's identity, 1923) $p_3 \leq 3$,
(3) (Joly, 1970) if F not formally real then $s_n < \infty, p_n \leq (n+1)s_n$,
(4) C_1-conjecture of S. Lang implies $s_n \leq n$,
(5) (B., 1982) all $p_n < \infty$, "reasonable" general bounds, in each case too large: e.g. $p_4 \leq 36$,
(6) (Choi, Lam, Prestel, Reznick, 1996) $p_4(\mathbb{R}(X)) \leq 6$.

New results:

(1) F formally real then $p_4(F) \leq 6$,
(2) $F = \mathbb{C}(X)$, elements $a \notin F^n + F^n$.

The genus-formula of Riemann-Hurwitz for $\mathbb{C}(X)/\mathbb{C}(f/g)$: $f, g \in \mathbb{C}[X]$, relatively prime, then

$$f/radf \cdot g/radg \cdot \prod_{\mu\neq 0}(f+\mu g)/rad(f+\mu g) = \text{ const } \cdot W(f,g).$$