Mean ergodic theorem for polynomial subsequences

Vladimir Müller

Lumini, 2015

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joint work with T. ter Elst

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(mean ergodic theorem): Let X be a reflexive Banach space and $T \in B(X)$ power bounded.

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(mean ergodic theorem): Let X be a reflexive Banach space and $T \in B(X)$ power bounded. Let $x \in X$. Then

$$\lim_{N\to\infty}\frac{1}{N}\sum_{n=1}^N T^n x$$

exists.

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Problem: Let (a_n) be an increasing sequence.

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$$\lim_{N\to\infty}\frac{1}{N}\sum_{n=1}^N T^{a_n}$$

exists? When $\frac{1}{N}\sum_{n=1}^{N} T^{a_n}$ converges to the "proper" limit?

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Akcoglu - Sucheston, Jones - Kuftinec, Lin (1971)

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Corollary

Let $T \in B(H)$ be a completely non-unitary contraction.

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not true for Banach spaces

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not true for Banach spaces true for ℓ_p , V.M., Tomilov (2007) open for L^p true for positive contractions on L^p , Below (1975) not true for power bounded Hilbert space operators, V.M., Tomilov (2007) $T^n \rightarrow 0$ (*WOT*) is necessary

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Vladimir Müller Mean ergodic theorem for polynomial subsequences

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Let $T \in B(H)$ be a polynomially bounded operator such that $T^n \rightarrow 0$ (WOT).

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Let $T \in B(H)$ be a polynomially bounded operator such that $T^n \to 0$ (WOT). Let $(a_n) \subset \mathbb{N}$ be a strictly increasing subsequence.

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$$N^{-1}\sum_{n=1}^{N}T^{a_n}
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 (SOT)

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$$N^{-1}\sum_{n=1}^{N}T^{a_n}\to 0\qquad(\text{SOT})$$

Theorem

Let $T \in B(H)$ be a power bounded operator of class $C_{.,1}$ such that $T^n \to 0$ (WOT).

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Blum - Eisenberg (1974)



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Blum -Eisenberg (1974) Let (a_n) be a strictly increasing sequence. The following statements are equivalent:

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Blum -Eisenberg (1974) Let (a_n) be a strictly increasing sequence. The following statements are equivalent: (i) $N^{-1} \sum_{n=1}^{N} T^{a_n}$ converges (SOT) for all Hilbert space contractions T; (ii) $N^{-1} \sum_{n=1}^{N} U^{a_n}$ converges (SOT)

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(van der Corput):

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(van der Corput): Let (u_n) be a bounded sequence in a Hilbert space.

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(van der Corput): Let (u_n) be a bounded sequence in a Hilbert space. For h = 0, 1, ... let

$$s_h = \limsup_{N \to \infty} \left| N^{-1} \sum_{n=1}^N \langle u_{n+h}, u_n \rangle \right|.$$

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Then

$$\lim_{N\to\infty}N^{-1}\sum_{n=1}^N u_n=0.$$

Let U be a unitary operator,

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Let U be a unitary operator, $\sigma_p(U) = \emptyset$.

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Let *U* be a unitary operator, $\sigma_p(U) = \emptyset$. Then

$$\lim_{N\to\infty}N^{-1}\sum_{n=1}^NU^{n^2}x=0$$

for each x.

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Let *U* be a unitary operator, $\sigma_p(U) = \emptyset$. Then

$$\lim_{N\to\infty} N^{-1} \sum_{n=1}^N U^{n^2} x = 0$$

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Corollary

Let U be a unitary operator,

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Corollary

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Vladimir Müller Mean ergodic theorem for polynomial subsequences

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Let $T \in B(H)$ be a contraction.



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$$\lim_{N\to\infty} N^{-1} \sum_{n=1}^N T^{p(n)} x$$

exists for each x.

The same is true for many reasonable sequences (a_n) .

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Let $T \in B(H)$ be power bounded, $x \in H$. Let (a_n) be strictly increasing convex sequence in \mathbb{N}

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Let $T \in B(H)$ be power bounded, $x \in H$. Let (a_n) be strictly increasing convex sequence in \mathbb{N} such that $\sup_n \frac{a_{2n}}{a_n} < \infty$.

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Let $T \in B(H)$ be power bounded, $x \in H$. Let (a_n) be strictly increasing convex sequence in \mathbb{N} such that $\sup_n \frac{a_{2n}}{a_n} < \infty$. Suppose that

$$\lim_{N\to\infty}N^{-1}\sum_{n=1}^N T^{a_{n+k}-a_n}x=0$$

for all $k \in \mathbb{N}$.

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Let $T \in B(H)$ be power bounded, $x \in H$.

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Let $T \in B(H)$ be power bounded, $x \in H$. Let p be a polynomial satisfying $p(\mathbb{N}) \subset \mathbb{N}$.

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$$\lim_{N\to\infty} N^{-1} \sum_{n=1}^N T^{p(n)} x$$

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$$\lim_{N\to\infty} N^{-1} \sum_{n=1}^N T^{p(n)} x$$

exists. If $\sigma_p(T) \cap \mathbb{T} = \emptyset$

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Let $T \in B(H)$ be power bounded, $x \in H$. Let p be a polynomial satisfying $p(\mathbb{N}) \subset \mathbb{N}$. Then the limit

$$\lim_{N\to\infty} N^{-1} \sum_{n=1}^N T^{p(n)} x$$

exists. If $\sigma_p(T) \cap \mathbb{T} = \emptyset$ then

$$\lim_{N\to\infty}N^{-1}\sum_{n=1}^N T^{p(n)}x=0.$$

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Questions.



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Questions.

(1) Is it possible to generalize this for other sequences?

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Questions.

- (1) Is it possible to generalize this for other sequences?
- (2) to some Banach spaces?

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Let $T \in B(H)$ be a power bounded operator on a Hilbert space H, $\sigma_p(T) \cap \mathbb{T} = \emptyset$.

Vladimir Müller Mean ergodic theorem for polynomial subsequences

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Let $T \in B(H)$ be a power bounded operator on a Hilbert space H, $\sigma_p(T) \cap \mathbb{T} = \emptyset$. Let $m \in \mathbb{N}_0$, let $f : [0, \infty) \to [0, \infty)$ be a function satisfying

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Let $T \in B(H)$ be a power bounded operator on a Hilbert space H, $\sigma_p(T) \cap \mathbb{T} = \emptyset$. Let $m \in \mathbb{N}_0$, let $f : [0, \infty) \to [0, \infty)$ be a function satisfying $f' > 0, f'' > 0, \cdots, f^{(m+1)} > 0$

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Let $T \in B(H)$ be a power bounded operator on a Hilbert space H, $\sigma_p(T) \cap \mathbb{T} = \emptyset$. Let $m \in \mathbb{N}_0$, let $f : [0, \infty) \to [0, \infty)$ be a function satisfying f' > 0, f'' > 0, \cdots , $f^{(m+1)} > 0$, $\sup\left\{\frac{f^{(m+1)}(s)}{f^{(m+1)}(t)} : 1 \le t \le s\right\} < \infty$,

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Let $T \in B(H)$ be a power bounded operator on a Hilbert space $H, \sigma_p(T) \cap \mathbb{T} = \emptyset$. Let $m \in \mathbb{N}_0$, let $f : [0, \infty) \to [0, \infty)$ be a function satisfying $f' > 0, f'' > 0, \cdots, f^{(m+1)} > 0$, $\sup\left\{\frac{f^{(m+1)}(s)}{f^{(m+1)}(t)} : 1 \le t \le s\right\} < \infty$, (for example, $f^{(m+2)} < 0$), and $\sup\left\{\frac{f^{(m)}(t)}{t^{f^{(m+1)}(t)}} : t \ge 1\right\} < \infty$.

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$$SOT - \lim_{N \to \infty} N^{-1} \sum_{n=1}^{N} T^{[f(n)]+h_n} = 0.$$

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Let $T \in B(H)$ be a power bounded operator on a Hilbert space H, $\sigma_p(T) \cap \mathbb{T} = \emptyset$.

Vladimir Müller Mean ergodic theorem for polynomial subsequences

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Let $T \in B(H)$ be a power bounded operator on a Hilbert space H, $\sigma_p(T) \cap \mathbb{T} = \emptyset$. Let $f(t) = \sum_{j=0}^k c_j t^{\alpha_j}$,

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Let $T \in B(H)$ be a power bounded operator on a Hilbert space $H, \sigma_p(T) \cap \mathbb{T} = \emptyset$. Let $f(t) = \sum_{j=0}^k c_j t^{\alpha_j}$, where $k \in \mathbb{N}_0$, $c_0, \ldots, c_k, \alpha_0, \ldots, \alpha_k \in \mathbb{R}$, $c_0 > 0$, $\alpha_0 > \max\{0, \alpha_1, \ldots, \alpha_k\}$.

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$$\lim_{N\to\infty} N^{-1} \sum_{n=1}^N T^{[f(n)]} x = 0$$

for all $x \in H$.

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$\lim_{N\to\infty} N^{-1} \sum_{n=1}^{N} T^{a_n}$ cannot exist in general if (a_n) is increasing too fast

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 $\lim_{N\to\infty} N^{-1} \sum_{n=1}^{N} T^{a_n}$ cannot exist in general if (a_n) is increasing too fast (exponentially)

 $\lim_{N\to\infty} N^{-1} \sum_{n=1}^{N} T^{a_n}$ cannot exist in general if (a_n) is increasing too fast (exponentially)

an example is the Foguel operator

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Let $1 , <math>X = L^{p}(\mu)$, let $T \in B(X)$ be a power bounded operator

Vladimir Müller Mean ergodic theorem for polynomial subsequences

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Let $1 , <math>X = L^{p}(\mu)$, let $T \in B(X)$ be a power bounded operator, $T \ge 0$, let $x \in X$, $x \ge 0$.

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Let $1 , <math>X = L^{p}(\mu)$, let $T \in B(X)$ be a power bounded operator, $T \ge 0$, let $x \in X$, $x \ge 0$. Let $(a_n)_{n=1}^{\infty}$ be a strictly increasing sequence of positive integers such that

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Let $1 , <math>X = L^{p}(\mu)$, let $T \in B(X)$ be a power bounded operator, $T \ge 0$, let $x \in X$, $x \ge 0$. Let $(a_{n})_{n=1}^{\infty}$ be a strictly increasing sequence of positive integers such that $\sup\left\{\frac{a_{2n}}{a_{n}}: n \in \mathbb{N}\right\} < \infty$.

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Let $1 , <math>X = L^{p}(\mu)$, let $T \in B(X)$ be a power bounded operator, $T \ge 0$, let $x \in X$, $x \ge 0$. Let $(a_{n})_{n=1}^{\infty}$ be a strictly increasing sequence of positive integers such that $\sup\left\{\frac{a_{2n}}{a_{n}}: n \in \mathbb{N}\right\} < \infty$. Suppose also that $\lim_{n\to\infty} d_{n} = \infty$

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Let $1 , <math>X = L^{p}(\mu)$, let $T \in B(X)$ be a power bounded operator, $T \ge 0$, let $x \in X$, $x \ge 0$. Let $(a_{n})_{n=1}^{\infty}$ be a strictly increasing sequence of positive integers such that $\sup\left\{\frac{a_{2n}}{a_{n}}: n \in \mathbb{N}\right\} < \infty$. Suppose also that $\lim_{n\to\infty} d_{n} = \infty$ and $D := \sup\{d_{j} - d_{k}: 1 \le j \le k\} < \infty$, where $d_{n} = a_{n+1} - a_{n}$.

Suppose that
$$\lim_{N\to\infty} N^{-1} \sum_{n=1}^{N} T^{a_{n+j}-a_n} x = 0$$
 $(j \in \mathbb{N})$

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Suppose that
$$\lim_{N \to \infty} N^{-1} \sum_{n=1}^{N} T^{a_{n+j}-a_n} x = 0$$
 $(j \in \mathbb{N})$

and
$$\lim_{N \to \infty} N^{-1} \sum_{n=1}^{N} T^{a_n - a_{n+j} + jD + (a_{N+j} - a_N)} x = 0$$
 $(j \in \mathbb{N}).$

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Then
$$\lim_{N\to\infty} N^{-1} \sum_{n=1}^N T^{a_n} x = 0$$

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Suppose that
$$\lim_{N \to \infty} N^{-1} \sum_{n=1}^{N} T^{a_{n+j}-a_n} x = 0$$
 $(j \in \mathbb{N})$

and
$$\lim_{N\to\infty} N^{-1} \sum_{n=1}^{N} T^{a_n-a_{n+j}+jD+(a_{N+j}-a_N)} x = 0$$
 $(j \in \mathbb{N}).$

Then
$$\lim_{N \to \infty} N^{-1} \sum_{n=1}^{N} T^{a_n} x = 0$$
 and $\lim_{N \to \infty} N^{-1} \sum_{n=1}^{N} T^{a_n - a_n} x = 0.$

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Let $X = L^{p}(\mu)$, $1 . Let <math>T \in B(X)$ be power bounded, $T \ge 0$, $\sigma_{p}(T) \cap \mathbb{T} = \emptyset$.

Vladimir Müller Mean ergodic theorem for polynomial subsequences

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Let $X = L^{p}(\mu)$, $1 . Let <math>T \in B(X)$ be power bounded, $T \ge 0$, $\sigma_{p}(T) \cap \mathbb{T} = \emptyset$. Let $f(t) = \sum_{j=0}^{k} c_{j}t^{\alpha_{j}}$, where $k \in \mathbb{N}_{0}$, $c_{0}, \ldots, c_{k}, \alpha_{0}, \ldots, \alpha_{k} \in \mathbb{R}$, $c_{0} > 0$, $\alpha_{0} > \max\{0, \alpha_{1}, \ldots, \alpha_{k}\}$.

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$$\lim_{N\to\infty} N^{-1} \sum_{n=1}^N T^{[f(n)]} x = 0$$

for all $x \in H$.

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Let $(T(t)_{t\geq 0})$ be a bounded strongly continuous semigroup on a Hilbert space H.

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Let $(T(t)_{t\geq 0})$ be a bounded strongly continuous semigroup on a Hilbert space H. Let $f(t) = \sum_{j=0}^{k} c_j t^{\alpha_j}$,

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Let $(T(t)_{t\geq 0}$ be a bounded strongly continuous semigroup on a Hilbert space H. Let $f(t) = \sum_{j=0}^{k} c_j t^{\alpha_j}$, where $c_0 > 0$, $c_j \in \mathbb{R}$, $\alpha_0 > \max\{0, \alpha_1, \dots, \alpha_k\}$. Suppose that $f \geq 0$.

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$$\lim_{N\to\infty} N^{-1} \int_0^N T_{f(t)} dt$$

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Let $(T(t)_{t\geq 0})$ be a bounded strongly continuous semigroup on a Hilbert space H. Let $f(t) = \sum_{j=0}^{k} c_j t^{\alpha_j}$, where $c_0 > 0$, $c_j \in \mathbb{R}$, $\alpha_0 > \max\{0, \alpha_1, \dots, \alpha_k\}$. Suppose that $f \geq 0$. Then

$$\lim_{N\to\infty} N^{-1} \int_0^N T_{f(t)} dt$$

exists in the strong operator topology

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Let $(T(t)_{t\geq 0})$ be a bounded strongly continuous semigroup on a Hilbert space H. Let $f(t) = \sum_{j=0}^{k} c_j t^{\alpha_j}$, where $c_0 > 0$, $c_j \in \mathbb{R}$, $\alpha_0 > \max\{0, \alpha_1, \dots, \alpha_k\}$. Suppose that $f \geq 0$. Then

$$\lim_{N\to\infty} N^{-1} \int_0^N T_{f(t)} dt$$

exists in the strong operator topology and is equal to the projection P onto the kernel of the generator of the semigroup (T(t))

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Let $(T(t)_{t\geq 0})$ be a bounded strongly continuous semigroup on a Hilbert space H. Let $f(t) = \sum_{j=0}^{k} c_j t^{\alpha_j}$, where $c_0 > 0$, $c_j \in \mathbb{R}$, $\alpha_0 > \max\{0, \alpha_1, \dots, \alpha_k\}$. Suppose that $f \geq 0$. Then

$$\lim_{N\to\infty} N^{-1} \int_0^N T_{f(t)} dt$$

exists in the strong operator topology and is equal to the projection *P* onto the kernel of the generator of the semigroup (T(t)) with ker $P = \overline{\bigcup_{\varepsilon > 0} (T - \varepsilon I) H}$.

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