Automorphic Forms: Advances and Applications
May 25 - 29, 2015

Speaker: **Scott Ahlgren** (University of Illinois)

Title: Algebraic and transcendental formulas for the smallest parts function

Abstract: We study the smallest parts function introduced by Andrews. The associated generating function forms a component of a natural mock modular form of weight 3/2 whose shadow is the Dedekind eta function. We obtain an exact formula and an algebraic formula for each value of the smallest parts function; these are analogues of the formulas of Rademacher and Bruinier-Ono for the ordinary partition function. The convergence of our expression is non-trivial; the proof relies on power savings estimates for weighted sums of generalized Kloosterman sums which follow from spectral methods.

Speaker: **Jan Bruinier** (Technische Universität Darmstadt)

Title: Classes of Heegner divisors and traces of singular moduli

Abstract: In parallel to the Gross-Kohnen-Zagier theorem, Zagier proved that the traces of the values of the j-function at CM points are the coefficients of a weakly holomorphic modular form of weight 3/2. Later this result was generalized in different directions and also put in the context of the theta correspondence. We recall these results and report on some newer aspects.

Speaker: **Miranda Cheng** (Universiteit van Amsterdam)

Title: Optimal Mock Jacobi Forms

Abstract: In this talk I will consider the space of optimal mock Jacobi forms of weight one. I define special elements symmetric optimal mock Jacobi forms and prove there are 39 of them, in one to one correspondence to the 39 genus zero non-Fricke Atkin-Lehner groups. They generate the space of optimal mock Jacobi forms in a natural way; 23 of them are singled out by a positivity condition and correspond to the 23 cases of umbral moonshine. Moreover, all Ramanujans mock theta functions arise from symmetric optimal mock Jacobi forms. Based on work with J. Duncan.

Speaker: **Nikolaos Diamantis** (University of Nottingham)

Title: Automorphic forms of general weights and cohomology

Abstract: We describe an analogue of the Eichler-Shimura cohomology for holomorphic modular forms of general weight. This analogue extends the cohomological setting for Maass cusp forms developed by Bruggeman, Lewis and Zagier, thus providing an unified framework for holomorphic and Maass forms. We show how the construction leads to a reinterpretation

of special values of derivatives of L-functions and discuss a link with quantum modular forms. (Joint work with R. Bruggeman and Yj. Choie)

SPEAKER: **Stephan Ehlen** (Mc Gill University)

TITLE: On two arithmetic theta lifts

ABSTRACT: In joint work with Siddarth Sankaran, we study two families of Green functions attached to special divisors on unitary and orthogonal Shimura varieties: the first family was introduced by Kudla and is closely related to the geometric theta series of Kudla and Millson, and the second was constructed by Bruinier as regularized theta lifts of harmonic weak Maass forms.

We show how to obtain Kudla's Green functions as regularized theta lifts of certain "truncated" Poincar series and indicate several applications, in particular to the generating series of arithmetic divisors defined by the two families of Green functions in the unitary case.

SPEAKER: **Sol Friedberg** (Boston College)

TITLE: Higher theta functions

ABSTRACT: Higher theta functions are the residues of Eisenstein series on covers of reductive groups. On the one hand, they generalize the Jacobi theta function, which comes from the double cover of GL_2. On the other, their Whittaker coefficients are not understood, even for higher covers of GL_2. In this talk I explain why one should expect a series of relations between the coefficients of theta functions on different groups and some new constructions which come close to establishing this, and explain how one may construct an automorphic form on the 4-fold cover of GL_2 with algebraic Fourier coefficients. This is based on on-going joint work with David Ginzburg.

SPEAKER: **Jens Funke** (University of Durham)

TITLE: Cohomolgical aspects of weakly holomorphic modular forms and periods

ABSTRACT: In this talk we give a simple cohomological identity between a weakly holomorphic form and a cusp form both of weight k obtained by applying certain differential operators to a given harmonic Maass form of weight $2-k$. We derive several consequences. In particular, we give a cohomological interpretation for the equality of periods of the two weight k forms in question.

SPEAKER: **Terry Gannon** (University of Alberta)

TITLE: Bounded denominators and p-curvature

ABSTRACT: Atkin-Swinnerton-Dyer observed in 1971 that a modular form for a finite-index subgroup of the modular group seems to have integral Fourier coefficients, only when it is a modular form for a congruence subgroup. It has been conjectured that vector-valued modular

forms for the modular group has integral coefficients only when a congruence subgroup is in the kernel of the multiplier. In my talk, I'll explain how p-curvature and Grothendieck's conjecture relate to this question.

SPEAKER: **Valery Gritsenko** (Université Lille I)

TITLE: Borcherds products and a new class of Lorentzian Kac-Moody Algebras

ABSTRACT: The denominator function of a Lorentzian Kac-Moody Algebra is a strongly reflective Borcherds product on the orthogonal group of signature $(2, n)$. "Strongly reflective" means that the divisors of the Borcherds product have multiplicity one and all of them are defined by reflections in the integral orthogonal group. The classification of such Borcherds products and the corresponding Lorentzian Kac-Moody Algebras is a very important and complicated problem. At the moment only two classes are obtained: Gritsenko-Nikulins class of the maximal algebras of rank 3 related to the Siegel paramodular forms of genus two (these algebras appeared in CHL-models) and Scheithauers class of the Borcherds products of singular weight. In this talk I present a new totally classified large family of the Borcherds products and the Kac-Moody algebras corresponding to the Weyl groups generated by all -2-reflections with positive Weyl vector in the hyperbolic lattices of any rank. This is my joint project with V. Nikulin. At the end I formulate some open problems which relate the modular forms from these three known classes.

SPEAKER: **Tomoyoshi Ibukiyama** (Osaka University)

TITLE: New theorems on Jacobi forms of general degree

ABSTRACT: I will give the following two results (as far as time allows, but (2) might be omitted).

(1) We show that the Taylor coefficients of Jacobi forms of general degree of any matrix index are essentially vector valued Siegel modular forms, using differential operators. This is applied to give several structure theorems in concrete cases.

(2) We give an isomorphism between Jacobi forms of general degree of index one and Siegel modular forms of half-integral weight for any level with or without character.

SPEAKER: **Winfried Kohnen** (Ruprecht-Karls-Universitt Heidelberg)

TITLE: A characterization of cusp forms by means of the growth of their Fourier coefficients"

ABSTRACT: We will discuss what bounds one has to put on the general Fourier coefficients of a modular form to guarantee that is already a cusp form. This question recently has attracted quite a bit of attention. We will present a rather general result, valid in the

context of arbitary Siegel modular forms. This is recent joint work with Böcherer (Feb. 2015).

Speaker: **Karl Mahlburg** (Louisiana State University)

Title: Automorphic forms and classical partition identities

Abstract: I will discuss recent progress in the analytic study of classical partition identities, including the famous "sum-product" formulas of Rogers-Ramanujan, Schur, and Capparelli. Such identities are rich in automorphic objects such as Jacobi theta functions, mock theta functions, and false theta functions. Furthermore, there are interesting connections to the combinatorics of multi-colored partitions, and the calculation of standard modules for Lie algebras and vertex operator theory.

Speaker: **Boris Pioline** (Université Paris 6 and CERN)

Title: A string theorist view point on the genus-two Kawazumi-Zhang invariant

Abstract: The genus-two Kawazumi-Zhang (KZ) invariant is a real-analytic modular function on the Siegel upper half-plane of degree two, which plays an important role in arithmetic geometry. In String theory, it appears as part of the integrand in two-loop four-graviton scattering amplitudes. With hindsight from String theory, I will show that the KZ invariant can be obtained as a generalized Borcherds lift from a weak Jacobi form of index 1 and weight -2. This implies that the KZ invariant is an eigenmode of the quadratic and quartic Casimir operators, and gives access to the full asymptotic expansion in all possible degeneration limits. It also reveals a mock-type holomorphic Siegel modular form underlying the KZ invariant. String theory amplitudes involves modular integrals of the KZ invariant (times lattice partition functions) on the Siegel upper half-plane, which provide new examples of automorphic objects on orthogonal Grassmannians, beyond the usual Langlands-Eisenstein series.

References: arXiv:1504.04182, arXiv:1405.6226

Speaker: **Ameya Pitale** (University of Oklahoma)

Title: Nearly holomorphic vector-valued Siegel modular forms of degree 2

Abstract: Shimura developed the theory of nearly holomorphic modular forms in the 1970s. One of the most important applications is to arithmeticity of special values of L-functions. In this talk, we present the recent joint work with Abhishek Saha and Ralf Schmidt on nearly holomorphic vector-valued Siegel modular forms of degree 2. In particular, we obtain a complete structure theorem stating that the space of nearly holomorphic modular forms is an orthogonal direct sum of forms obtained by applying certain differential operators to vector-valued holomorphic Siegel modular forms. The main step is to obtain the characterization of such modular forms in the automorphic world and then use representation theoretic methods applied to the lowest weight representations of $Sp(4,\mathbb{R})$ to get the structure theorem. This allows us to obtain complete results in all cases, a significant

improvement from Shimura's result. The differential operators involved come from Lie algebra elements and we write them down explicitly in terms of coordinates in the Siegel upper half space. As an application, we obtain algebraicity results on ratios on Petersson norms of such modular forms.

Speaker: **A. Raghuram** (IISER Pune)

Title: Endoscopy and the cohomology of arithmetic groups

Abstract: Given an integer k, taken to be at least 2, it is well-known that there are holomorphic cusp forms of weight k for some subgroup of $SL(2,\mathbb{Z})$. In this talk, I will discuss some higher-dimensional analogues, which beg the question: given a connected reductive group G, and a finite-dimensional representation V of G, is the cuspidal cohomology of G with coefficients in V nonzero? After introducing the context, and discussing some general themes, I will present some results, obtained in an ongoing collaboration with Chandrasheel Bhagwat, that give an endoscopic construction of nonzero cuspidal cohomology classes when G is $GL(n)$ over a totally real field.

Speaker: **Larry Rolen** (Universität zu Köln)

Title: Mock theta functions and quantum modular forms

Abstract: In this talk, I will describe several related recent results related to mock theta functions, which are functions described by the Indian mathematician Ramanujan shortly before his death in 1920. These functions have very recently been understood in a modern framework thanks to the work of Zwegers and Bruinier-Funke. Here, we will revisit the original writings of Ramanujan and look at his original conception of these functions, which gives rise to a surprising picture connecting important objects such as generating functions in combinatorics and so-called quantum modular forms.

Speaker: **Nils Skoruppa** (Universität Siegen)

Title: Explicit methods in the theory of Jacobi forms

Abstract: In recent years we observed an increasing interest into an extension of the classical theory of Jacobi forms, in particular, into Jacobi forms with index of higher rank and Jacobi forms over number fields. We report about recent progress in the arithmetic theory of these types of Jacobi forms with an emphasis of explicit methods and examples. The talk is based on recent results of joint work with A. Ajouz, H. Boylan, S. Hayashida, and F. Strömberg.

Speaker: **Harold Stark** (University of California at San Diego)

Title: The construction of modular forms

Abstract: TBA

Speaker: **Martin Westerholt-Raum** (MPIM Bonn)

Title: Is there a Siegel analogue of E_2?

Abstract: The almost holomorphic elliptic modular form E_2 undoubtedly is the easiest harmonic weak Maass form with image under the lowering operator the constant function 1. We show that for Siegel modular forms there is no analogue of E_2, and in particular, classify all almost holomorphic Siegel modular forms. In degree greater than 1, they arise without exception from derivatives of holomorphic ones.

We introduce almost meromorphic Siegel modular forms. In degree 2 we describe an almost meromorphic preimage under the lowering operator of the constant Siegel modular form 1. The Ramanujan differential equation for E_2 generalizes to this setting. In topological string theory, it can be interpreted as a propagator identity for genus 2 curves.

Speaker: **Wadim Zudilin** (University of Newcastle)

Title: Modular parametrizations of 4th order linear differential operators and their applications in number theory

Abstract: There are families of examples of 4^{th} order Picard-Fuchs differential operators that appear in several arithmetic contexts (e.g., formulas for $1/\pi$ and Mahler measures) and can be realized as symmetric products of 2^{nd} order operators. I will outline some known examples and also highlight certain wanted cases, together with their (potential) applications. Parts of the talk are based on joint work with Armin Straub and James Wan.

Speaker: **Sander Zwegers** (Universität zu Köln)

Title: Fourier coefficients of meromorphic Jacobi forms

Abstract: Fourier coefficients of meromorphic Jacobi forms show up in, for example, the study of mock theta functions, quantum black holes and Kac-Wakimoto characters. In the case of positive index, it was previously shown that they are the holomorphic parts of vector-valued almost harmonic Maass forms. In this talk, we give an alternative characterization of these objects by applying the Maass lowering operator to the completions of the Fourier coefficients. Further, we'll also describe the relation of Fourier coefficients of negative index Jacobi forms to partial theta functions.