Cohomological Methods in the Theory of Algebraic Groups
August 31 - September 4, 2015

Paul Balmer (joint work with Beren Sanders): The spectrum of the equivariant stable homotopy category.

After recalling why one cares about the spectrum of a tensor-triangulated category, we shall discuss the spectrum of the equivariant stable homotopy category of a finite group. We shall describe all the primes, i.e. all the points in this spectrum, and then discuss the topology.

Eva Bayer-Fluckiger: Embeddings of maximal tori in classical groups and explicit Brauer-Manin obstruction.

This is a joint work with Parimala and Tingyu Lee. Embeddings of maximal tori into classical groups over global fields of characteristic $\neq 2$ are the subject matter of several recent papers (for instance by Prasad and Rapinchuk, Fiori, Lee), with special attention to the Hasse principle. The aim of this talk is to give a complete criterion for the Hasse principle to hold , as well as necessary and sufficient conditions for an embedding to exist. The embedding problem will be described in terms of embeddings of étale algebras with involution into central simple algebras with involution.

Patrick Brosnan: Motivic decompositions of varieties of unseparated flags after Srinivasan.

Suppose G is a reductive group over a perfect field k acting transitively on a geometrically connected, projective variety X. If k has characteristic 0, then X is (almost, by definition) a projective homogenous variety, geometrically isomorphic to G/P for some parabolic P. However, this is no longer true if the characteristic of k is positive. In that case, X is geometrically isomorphic to G/Q where Q is a subgroup scheme of G which may not be reduced. The varieties X, which are easily seen to smooth and projective, are called varieties of unseparated flags. I report on work of my student S. Srinivasan who has proved that the Rost nilpotence principle holds for varieties of unseparted flags. She has used this result along with Karpenko's theory of upper motives to show that, when G is inner, the motive of such a variety X is isomorphic to the motive of an associated projective homogeneous variety.

Jean-Louis Colliot- Thélène: Troisième groupe de cohomologie non ramifiée et variétés rationnellement connexes.

Le troisième groupe de cohomologie non ramifiée d'une variété lisse, à coefficients dans les racines de l'unité tordues deux fois, intervient dans plusieurs articles récents, en particulier en relation avec le groupe de Chow de codimension 2. On fera un tour d'horizon : espaces homogènes de groupes algébriques linéaires; variétés rationnellement connexes sur les complexes; images d'applications cycle sur les complexes, sur un corps fini, sur un corps de nombres.

Philippe Gille: Cohomologie non abélienne non ramifiée.

L'exposé est issu d'un travail en cours en collaboration avec V. Chernousov et A. Pianzola (Edmonton). En procédant par analogie avec le groupe de Brauer non ramifié d'une variété algébrique X, Colliot-Thélène a défini le sous-ensemble des classes de cohomologie non ramifiées de l'ensemble de cohomologie $H^1(k(X), G)$ pour un groupe algébrique linéaire G. Nous discuterons ce thème notamment dans le cas où X est un tore déployé en lien avec la théorie des algèbres de Lie toroidales tordues.

Boris Kunyavskii: Semisimple stably Cayley groups.

In 1846, Arthur Cayley defined a correspondence between orthogonal matrices of determinant one and skew-symmetric matrices. This observation was a starting point of a long (and yet unfinished) story. In the talk we will overview its highlights, with a focus on the achievements obtained during the past decade and some open problems.

Marc Levine: The rational motivic sphere spectrum and motivic Serre finiteness.

After inverting 2, the motivic sphere spectrum splits into a plus part and a minus part with respect to a certain natural involution. Cisinsky and Déglise have shown that, with rational coefficients, the plus part is given by rational motivic cohomlogy. With Ananyevskiy and Panin, we have computed the minus part with rational coefficients as being given by rational Witt-theory. In particular, this shows that the rational bi-graded homotopy sheaves of the minus sphere are concentrated in bi-degree (n,n). This may be rephrased as saying that the graded homotopy sheaves of the minus sphere in strictly positive topological degree are torsion. Combined with the result of Cisinski-Déglise mentioned above, this shows that the graded homotopy sheaves of the sphere spectrum in strictly positive topological degree and non-negative Tate degree are torsion, an analog of the classical theorem of Serre, that the stable homotopy groups of spheres in strictly positive degree are finite.

Ivan Panin: On the Grothendieck–Serre conjecture concerning principal G-bundles over reductive group schemes.

Let R be a regular local ring, containing a field. Let G be a reductive group scheme over R. Recently it was proved that a principal G-bundle over R is trivial, if it is trivial over the fraction field of R. In other words, if K is the fraction field of R, then the map of non-abelian cohomology pointed sets $H^1_{\text{et}}(R, G) \to H^1_{\text{et}}(K, G)$ induced by the inclusion of R into K, has the trivial kernel. That proves in positive the conjecture from the title. We will give a survey on the topic.

Raman Parimala: A Hasse principle for simply connected groups over function fields of p-adic curves.

Hasse principle for rational points on principal homogeneous spaces under semisimple simply connected linear algebraic groups over number fields is a theorem of Kneser, Harder and Chernousov. A similar Hasse principle for function fields of p-adic curves was posed as a conjecture by Colliot-Thelene, Parimala and Suresh. We discuss the developments and recent progress towards this conjecture for groups of inner type A_n. (Joint work with R. Preeti and V. Suresh).

Alena Pirutka: On examples of varieties that are not stably rational.

A variety X is stably rational if a product of X and some projective space is rational. There exists examples of stably rational non rational complex varieties. In this talk we will discuss recent series of examples of varieties, which are not stably rational and not even retract rational. The proofs involve studying the properties of Chow groups of zero-cycles and the diagonal decomposition. As concrete examples, we will discuss some quartic double solids (C. Voisin), quartic threefolds (a joint work with Colliot-Thélène), some hypersurfaces (Totaro) and others.

Anne Quéguiner-Mathieu: Arason Invariant for orthogonal involutions.

Using the Rost invariant for torsors under Spin groups, one may define an analogue of the Arason invariant for certain hermitian forms and orthogonal involutions. This talk will survey what is known on this question, focusing on the degree 8 and degree 12 cases, which were explored in joint works with Jean-Pierre Tignol.

Zinovy Reichstein: The rationality problem for forms of moduli spaces of stable marked curves.

Let $\overline{M_{g,n}}$ be the moduli space of stable curves of genus g with n marked points. It is a classical problem in algebraic geometry to determine which of these spaces are rational over $\mathbb{C}$. In this talk, based on joint work with Mathieu Florence, I will address the rationality problem for twisted forms of $\overline{M_{g,n}}$. Twisted forms of $\overline{M_{g,n}}$ are of interest because they shed light on the arithmetic geometry of $\overline{M_{g,n}}$, and because they are coarse moduli spaces for natural moduli problems in their own right. A classical result of Yu. I. Manin and P. Swinnerton-Dyer asserts that every form of $\overline{M_{0,5}}$ is rational. (Recall that the F-forms $\overline{M_{0,5}}$ are precisely the del Pezzo surfaces of degree 5 defined over F.) Mathieu Florence and I have proved the following generalization of this result.

Let $n \geq 5$ is an integer, and F is an infinite field of characteristic $\neq 2$.

(a) If n is odd, then every twisted F-form of $\overline{M_{0,n}}$ is rational over F.

(b) If n is even, there exists a field extension F/k and a twisted F-form of $\overline{M_{0,n}}$ which is unirational but not retract rational over F.

We also have similar results for forms of $\overline{M_{g,n}}$, where $g \leq 5$ (for small n). In the talk, I will survey the geometric results we need about $\overline{M_{g,n}}$, explain how our problem reduces to the Noether problem for certain twisted goups, and how this Noether problem can (sometimes) be solved.

David Saltman: Division algebras and separable subfields.

If D/F is a division algebra (finite dimensional) and K/F is a Galois maximal subfield then this forces D/F to be a crossed product, which is a strong constraint on the algebra structure of D. Generic separable K/F have Galois group the symmetric group S_n (i.e. the Galois group of its Galois cclosure) but one can ask whether there are algebraic consequences of D/F having maximal K/F where the Galois group of K/F is smaller than S_n (but bigger than order n). With the machinery we develop we answer questions about the asymptotic behavior of $(KaK)^m$ and characterize when aK consists entirely of n power central elements. We also use this machinery to understand and in a sense generalize the old proof that dihedral algebras are cyclic. This is joint work with Eli Matzri, Louis Rowen, and Uzi Vishne.

Alexander Sivatski: The principal results of Alexander Merkurjev on central simple algebras over fields (survey).

The talk is devoted to the major contribution of A.Merkurjev to the theory of central simple algebras over fields. We are going to discuss the following problems:

1) The norm residue homomorphism theorem for $p = 2$, and for an arbitrary prime p.

2) The index reduction theorem for quadrics and the construction of fields with arbitrary even U-invariant.

3) The lower bound for the essential p-dimension of the group PGL_n.

4) General nontriviality of the reduced Whitehead group for central simple algebras of nonsquarefree index. Time permitted, we consider certain open questions and conjectures.

Andrei Suslin: The group K_2 of a biquaternion algebra.

For a central simple algebra D, set

$$\overline{K}_i(D) = Coker(\oplus_E K_i(E) \to K_i(D))$$

where direct sum is taken over all splitting fields of D. Clearly $\overline{K}_1(D)$ is trivial for any D. Vanishing of $\overline{K}_2(D)$ for algebras of squarefree degree was proved by Merkurjev and Suslin in 1981. Whether $\overline{K}_2(D)$ is trivial for any algebra is still unclear. In this talk we discuss a formula which, in the case of a biquaternion algebra, relates $\overline{K}_2(D) \oplus SK_2(D)$ to Galois cohomology of the field F.

Jean-Pierre Tignol: Outer automorphisms of algebraic groups.

For an absolutely simple linear algebraic group that is simply connected or adjoint, the action of the automorphism group of the Dynkin diagram on the Tits class provides an obstruction to the existence of outer automorphisms. This talk will report on a joint work with Anne Quéguiner-Mathieu, in which we give examples where outer automorphisms do not exist, even though the Tits class obstruction vanishes. This settles in the negative a conjecture of Garibaldi-Petersson.

Burt Totaro: Decomposition of the diagonal, and applications.

Decomposition of the diagonal is a basic method in the theory of algebraic cycles. The method relates the birational geometry of a variety to properties of the Chow groups. One recent application is that the Chow ring of a finite group can depend nontrivially on the base field, even for fields containing the algebraic closure of Q. Another application is that a very general complex hypersurface in P^{n+1} of degree at least about 2n/3 is not stably rational.

Alexander Vishik: Subtle Stiefel-Whitney classes and the J-invariant of quadrics.

I will discuss the new ?subtle? version of Stiefel-Whitney classes introduced by Alexander Smirnov and me. In contrast to the classical classes of Delzant and Milnor, our classes see the powers of the fundamental ideal, as well as the Arason invariant and its higher analogues, and permit to describe the motives of the torsor and the highest Grassmannian associated to a quadratic form. I will consider in more details the relation of these classes to the J-invariant of quadrics. This invariant defined in terms of rationality of the Chow group elements of the highest Grassmannian contains the most basic qualitative information on a quadric.